AUTOMATIC CONTROL

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June 1 2003

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FOREWORD

Automatic control theory is evolved from its first formulation for more than fifty years in the last half of the past century. Frequency domain was the first representation of dynamical systems used for analysis and design of feedback control systems, and from that period formalisms and criteria introduced by Bode and Nyquist become classical. Successively state variable representation was introduced and paved the road to optimum control, Linear Quadratic optimum control, until the robust control based on H_∞ norm of the present days.

It is surprising that, in spite of the richness of the modern results and the well known limitations of the frequency domain, it is customary to start the curriculum on automation with a first course of automatic control that still maintain the classical setting older than fifty years (in reality the very first is a course of system theory but it mostly copes with modeling dynamical systems). To the more modern contents are devoted successive courses, almost considering them as a specialization. Moreover non-specialists, still continue to use classical frequency based techniques, even if is commonly accepted their shortcoming outside the SISO systems.

Reason for that are dual:

From the one side frequency domain is a custom culture in the electronic field where most of the control practitioners come from, moreover, paper and pencil Bode and Nyquist techniques are promptly catch by neophytes, and offer immediate results.

From the other side modern techniques, based on state variable representation in the present formulation look more like mathematical exercises than engineering instruments, are fairly complex requiring a strong background on algebra, and finally lack robust and easy to use numerical algorithms for synthesis. Tools available are still difficult to use.

The objective of this book is ambitious: bring modern techniques based on robust control in the first course of automatic control substituting frequency domain methods. We intend to achieve this result by emphasizing exclusively the intuitive aspects present in the available approach, by reducing algebra results, and especially (quadratic) optimization, as pure implementation aspects hidden in the numerical algorithms, in the same way that people scarcely know numerical algorithms for computing matrix eigenvalues even if they currently use them.

To be useful, design techniques must be accompanied by specific numerical tools for synthesis easy to use and reliable in order to achieved immediate and complete results.

Obviously the approach starts directly from MIMO systems. SISO systems in the frequency domain are exclusively introduced as a special case to help to visualize concepts such as loop function, sensitivity function, frequency band and to allow to see the fre-

quency interpretation of the H_∞ norm, but they are immediately transferred for any practical purpose to the state space.

The steps followed for the development of the program are fours:

1. Formalisms and definitions
   - The concept of input-output operator of asymptotically stable linear invariant systems must be understood, as well as its H_2 e H_∞ norms;
   - Additive, multiplicative noise (process uncertainty, in general) and control specification are added to the model employing the extended system representation, where input, disturbance, objective and measure are evidenced;
   - With the extended system, coping with uncertainty, the concept of robust stability is introduced through the small gain theorem. This theorem substitutes the Nyquist criterion;
   - From additive disturbance descends the concept of performance that substitute classical results on disturbance sensitivity in the closed loop;
   - Block diagrams give the way to linear fractional transformations.
2. The measure of achieved performances
   - Observability and controllability gramians will play an important role to assign a measure to state observability and controllability; to compute them very robust numerical solutions of Lyapunov equations are proposed;
   - From the gramians directly descend the H_2 norm of the closed loop dynamical operator;
   - A little less directly, through the worst case disturbances, the definition of upper bound of the H_∞ norm is also given;
3. Observer and state feedback
   - The two problems are presented exploiting duality and the definition of transpose system;
   - Output feedback through concatenation of observer and state feedback and the separation theorem are a background from system theory;
   - In the realm of the separation theorem we show how observer-state feedback structures allow to achieve generic disturbance sensitivity reductions and robustness improvement by directly operating on the gramians; in particular, we teach how to build monotonically decreasing sequences of gramians starting from a stabilizing solution; this result is provided by one of the theorems related to the algebraic Riccati equation, even if this equation is never explicitly mentioned;
4. Output feedback synthesis as two steps in cascade: design of the observer and from this the state feedback.
   - The envelop of control specifications results in the extended system for the design, and the assignment of a bound for the H_∞ closed loop norm for guaranteeing the achievement of a desired level of performance/robustness;
   - The observer is designed first by guaranteeing an assigned upper bound of the H_∞ norm operator from disturbances to the errors of estimating the objec-

tives. This is achieved by making small a measure of a particular controllability gramian of the estimation error operator;

- This observer in the worst disturbance case results in a model for designing a state feedback, where the estimation errors of the measures play the role of input disturbances, the objectives are the estimated objectives and the state estimates are directly available for feedback ; we call this the "*black box model*" for our control problem, as the closed loop performance it achieves, under certain conditions, guarantees the one when the observer-state feedback will be applied, as an output feedback, to the real plant. State feedback design is done operating on a measure of the closed loop observability gramian with duality with respect to the previous step;
- The performance of the overall control, obviously, cannot be lower than the errors in estimating the objectives offered by the estimator – hence, if a state feedback of the black box model can be found with a H_∞ norm performance not worse than the bound achieved on the estimation error, the closed loop is nominally stable and this bound is also guaranteed for the performance achieved by applying the observer-state feedback to control the plant; this is our interpretation of the different theorems at the basis of the theory of robust control with H_∞ norm.

This book doesn't offer new results on robust control theory, compared with the results already available on the literature. Viceversa, literature results are filtered, and for the few strictly needed, simple and intuitive proofs are given, even if not completely rigorous.

The true contribution we offer is to link punctually design techniques to the complete set of engineering specifications, the designer is used to in classical feedback control approaches. We adopt from the beginning the concept of gramians and we develop it to the analysis, evaluation of performances, up to the synthesis that is fully based on gramian computations. For this we could almost call this book Robust Control – an approach based on gramians.