Chapter 9

CONTROL OF A HYDRAULIC MOTOR

This chapter is devoted to the development of one classical example. We continue here to maintain a connection with classical approaches, with the objective to conduct a comparison between classical and modern design techniques.
The example is taken from the control of hydraulic joints of an industrial robot. The peculiarity of the problem is, precisely, given by the characteristics of the hydraulic motor.

9.1 Modeling a hydraulic motor

The basis of a hydraulic motor is a cylinder moved by oil maintained at high pressure through a compressor and controlled by a valve.

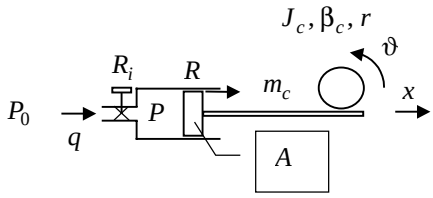

Fig. 9.1 Hydraulic motor

The equation of the dynamic behavior of the system are the following:
Valve and its linearization

$$q^2 = \frac{P_0 - P}{R_i}; \quad q = k \cdot i \cdot \sqrt{1 - \frac{P}{P_0}}$$

$$\delta q = k_i \cdot \delta i - k_P \cdot \delta P$$

$$k_i = k \cdot \sqrt{1 \pm \frac{P_n}{P_0}}, \quad k_P = \frac{k \cdot |i_n|}{2P_0 \cdot \sqrt{1 \pm \frac{P_n}{P_0}}} \tag{9.1}$$

$$P_0 = 2 \cdot 10^4 \frac{N}{m^2}, \quad P_n = 10^4 \frac{N}{m^2}, \quad q_n = 10^{-2} \frac{m^3}{s}, \quad i_n = 0.1A, \quad k = 0.1 \frac{m^3}{s \cdot A}$$

The range of variation of the variables present in the model are

$$\left| \frac{P_n}{P_0} \right| \le 0.8, \quad |i| \le 0.1A$$

as a consequence the variations of the linearization parameters are

$$\tilde{k}_p = 0 \div 0.55 \cdot 10^{-6}, \quad \tilde{k}_i = 0.045 \div 0.134$$

Hydraulic (oil compressibility and spilling)

$$q = C \cdot \dot{P} + \frac{P}{R} + A \cdot \dot{x}$$

$$C = 10^{-8} \frac{m^5}{N}, \quad R = 10^{10} \frac{N \cdot s}{m^5}$$

Mechanics

$$A \cdot P = m_c \cdot \ddot{x} + \frac{J_c}{r} \cdot \ddot{\vartheta} + \frac{\beta_c}{r} \cdot \dot{\vartheta}$$

$$m_c = 65\,kg, \quad J_c = 2\,kg \cdot m^2, \quad r = 0.25\,m, \quad \beta_c = 25 \frac{N \cdot m \cdot s}{rad}$$

$$m = m_c + \frac{J_c}{r^2}, \quad \beta = \frac{\beta_c}{r^2}$$

$$A = 0.1m^2, \quad m = 97\,kg, \quad \beta = 400 \frac{N \cdot s}{m}$$

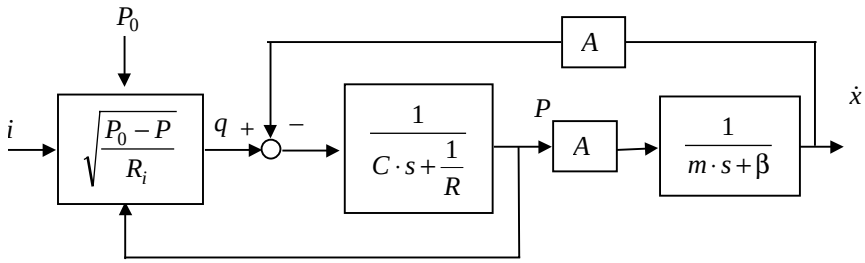

Fig. 9.2 Block diagram of the non-linear system

After linearization the model becomes

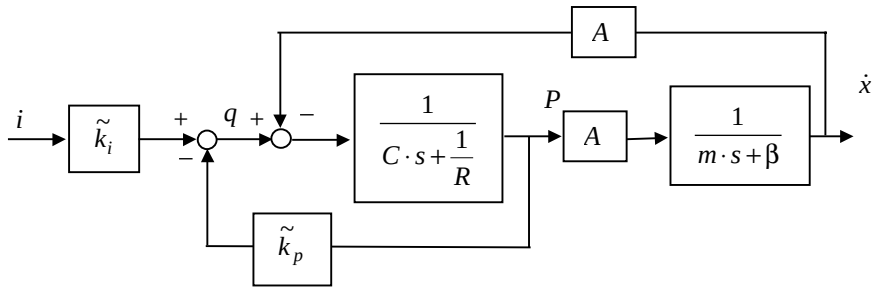

Fig. 9.3 Block diagram of the linearized model

As an exercise, in the following we will develop two different interpretations of the non-linearity in terms of uncertainty. The first interpretation is more schematic but allow to be approached with classical SISO techniques. The second more accurate can be handled with MIMO approaches, only.

9.1.1 Uncertainty description using transfer functions

The transfer function of the linearized model resulting from figure 9.3 is

$$\tilde{G}(s) = \frac{10^5 \cdot \tilde{k}_i}{s \cdot \left(s^2 + (4.1 + \tilde{\lambda}) \cdot s + 10^4 + 4.1 \cdot \tilde{\lambda}\right)}, \quad \tilde{\lambda} = 0.01 + 10^8 \cdot \tilde{k}_p = 0.01 \div 55 \cdot$$

The two parameters are assumed to change independently, introducing in the model two sources of uncertainty:

$$\tilde{G}(s) = \frac{8950(1+0.49\cdot\Delta_2)}{s\cdot\left(s^2+68.5\cdot s+10113\right)+s\cdot(27.5\cdot s+113)\cdot\Delta_1}.$$

$$\tilde{G}(s) = G(s)\frac{1+W_{22}\cdot\Delta_2}{1+W_{21}\cdot\Delta_1}$$

with

$$G(s) = \frac{8950}{s\cdot\left(s^2+68.5\cdot s+10113\right)}, \tag{9.2}$$

$$W_{22} = 0.49, \quad W_{21} = \frac{27.5(s+4.1)}{s^2+68.5\cdot s+10113}. \tag{9.3}$$

We remember that in this situation the condition of robust stability requires

$$\left|W_{21}\cdot S(j\omega)\right| + \left|W_{22}\cdot G_{eq1}(j\omega)\right| < 1, \quad \forall\omega. \tag{9.4}$$

9.1.2 Uncertainty described in the state variable domain

A more realistic description of the uncertainty introduced by non-linearity requires the adoption of a state variable representation in a multivariable context.
Let consider, again, the two linearization parameters, introducing two new variables $\tilde{d}_1$, and $\tilde{d}_2$, and rewrite them as

$$k_i = k\cdot\tilde{d}_1, \quad k_P = \frac{k\cdot\tilde{d}_2}{2P_0\cdot\tilde{d}_1} \tag{9.5}$$

with

$$\tilde{d}_1 = \sqrt{1\pm\frac{P_n}{P_0}}, \quad \tilde{d}_2 = |i_n| \tag{9.6}$$

At the variation of P_n and i_n the parameters assume the following form

$$k_i = a(1+\delta_1\Delta_1), \quad k_P = \frac{b(1+\delta_2\Delta_2)}{(1+\delta_1\Delta_1)} \tag{9.7}$$

with numerical values $a = 0.089,\ \delta_1 = 0.49,\ b = 1.4\cdot 10^{-7},\ \delta_2 = 1$.
The block diagram of the extended system encompassing uncertainty and some loop specifications is presented in the following figure

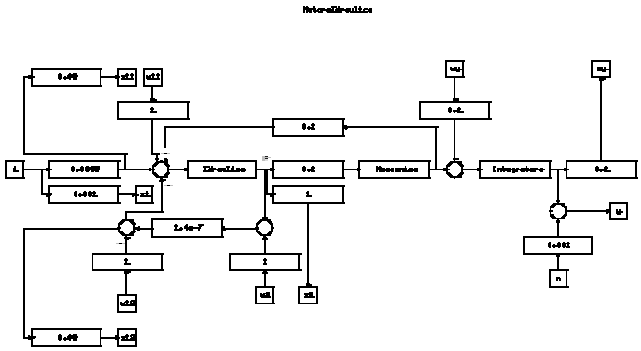

Fig. 9.4 Extended system of the control problem of a hydraulic motor

9.2 Requirements

The requirements for the position control of a joint controlled by the hydraulic motor are the following:
reference-output relationship

- 1) rise time to a step $t_s \leq 0.3$;
- 2) maximum overshoot $\hat{s} \leq 0.2$;
- 3) tracking error to a ramp $e_\infty = 0$.

Closed loop sensitivity
Let the relative sensitivity of the closed loop transfer function to the variations Δ_2 be:

$$4)\ \left|\frac{\Delta G_{eq}}{G_{eq}}\right| \leq 0.1 .$$

Sensitivity to additive disturbances
The input amplifier is subject to input offsets, at steady state the effect on the controlled output must be zero.

Control activity
Transducers for measuring position and velocity are available. They are characterized by independent white noise with standard deviation $\sigma^2 = 10^{-6}$.

The control activity, generated by these noises, measured by the norm $\|u\|_S$ of the power spectrum must be limited:

$$5)\ \|u\|_S^2 = \sup_w \left|G_{equ}\right|^2 \cdot \sigma^2 \le 10^{-4}\cdot$$

This requirement translates in the following condition

$$\left|W_3 \cdot G_{equ}(j\omega)\right| < 1, \quad \forall\omega; \quad W_3 = 0.1\cdot \tag{9.8}$$

that can be also written as

$$\left|\overline{W_3} \cdot G_{eq1}(j\omega)\right| < 1, \quad \forall\omega; \quad \overline{W_3} = \frac{0.1}{G}\ . \tag{9.9}$$

9.3 Loop shaping in the frequency

We note preliminarily that requirements of sensitivity of the closed loop function require a two degree of freedom design. We consider the loop characteristics, first, then we satisfy reference-output requirements.
By necessity with this approach we are constrained to adopt the first form of representation of the uncertainty.

9.3.1 Closing the loop

Performances

The plant has already a pole in zero, requirement 3) of error zero to the ramp in one degree of freedom design would require to add a further pole in zero in the loop. With a two degree of freedom design previous requirement can be achieved exploiting the second degree of freedom, acting on the feedforward filter without increasing the weight of the loop dynamics.
However, a constant noise on the input is present. The need to guarantee zero output error in steady state, and not the previous requirement on reference tracking, asks for one more pole in zero in the loop.
The closed loop reference-output frequency band because of requirement 1) on the rising time is $\omega_B \approx 10\,\frac{rad}{s}$. At that frequency requirement 4) imposes the following constraint on the loop sensitivity:

$$\left|W_1(j\omega)\right| = 10\cdot\left|W_{22}(j\omega)\right| = 4.9, \quad \omega = 10\cdot$$

From the two previous conditions we assume the following output weighting function

$$W_1(s) = \frac{490 \cdot 400}{s^2(s^2 + 28 \cdot s + 400)}. \tag{9.10}$$

The second pair of poles in the function is justified by the fact that at frequencies greater than the band we are not interested to impose further constraints to the sensitivity, sending the gain of the weight fast to zero.
Let start the design to satisfy performance requirements.
Compensator – low frequencies
At the low frequencies we assign to our compensator the following function

$$G_{ci}(s) = \frac{572}{s}.$$

Compensator – middle frequencies
We add to the compensator a lead action to guarantee the needed stability margins

$$G_{ca}(s) = \frac{5(s+20)}{s+100}.$$

We can verify that with this compensator the system is stable in nominal conditions and satisfy sensitivity requirements

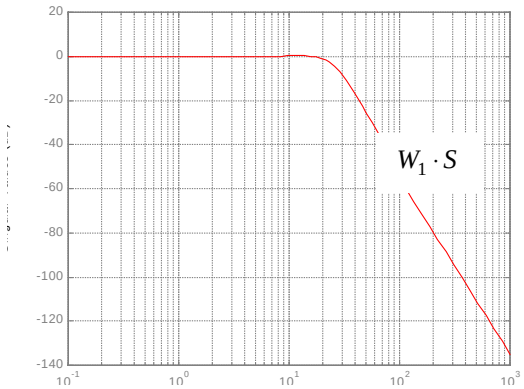

Fig. 9.5 Performance index

Robust stability

We check, now, robust stability conditions represented by (9.4).

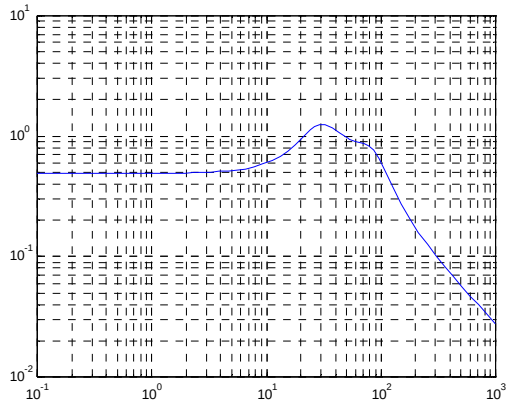

Fig. 9.6 Robust stability index

Robust stability is not guaranteed. Responsible for this is a poor phase margin. We substitute the original lead action with a stronger one.

$$G_{ca}(s) = \frac{8(s+20)}{s+160}$$

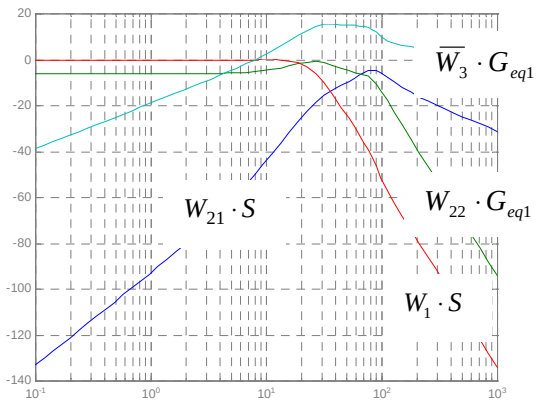

Fig. 9.7 Principal closed loop characteristics

The achieved result guarantees robust stability and nominal performances, robust performances, viceversa, are not satisfied. Nevertheless, all basic elements are now evidenced, so that with trial and errors, the design can be improved, e.g. adding a second lead action. Finally observe the index of control activity. This requirement is not satis-

fied. In fact, it was not considered in this design. It is doubt, without introducing other measurements, beyond the output, if it is possible to reduce the control activity. We will see if the synthesis will be able to improve the results.

9.3.2 Reference-output relationship

Requirements 1), 2) e 3) define the desired closed loop transfer function, of the type proposed in (6.6)

$$G_{eqdes1} = \frac{53.9(s+0.535)}{(s^2+8.4s+49)(s+0.59)}.$$

To guarantee steady state tracking we have an unacceptable rise time.
An alternative function is the following

$$G_{eqdes2} = \frac{16(s+3.5)}{3.5(s^2+4.8s+16)}. \tag{9.11}$$

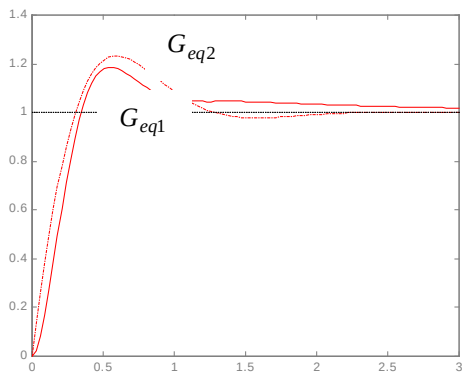

Fig. 9.8 Step response for two different transfer functions

The process follows the classical path of a two degree of freedom design.
A possible feedforward filter with the previous loop compensation and a desired closed loop function (9.11) is the following

$$G_f = \frac{0.11715(s+3.5)(s^2+28.12s+780.5)}{(s+20)(s^2+4.8s+16)}. \tag{9.12}$$

9.4 Synthesis in state variable

We apply state variable synthesis adopting both uncertainty models presented in the previous sections.
The first example replicates exactly the classical design, the second one with the new uncertainty model has more restrictive conditions, so it will be not directly comparable with the classical design.

9.4.1 Model 1

Interpretation in state variable form of the model (9.2) and requirements (9.10) carries to the following extended system

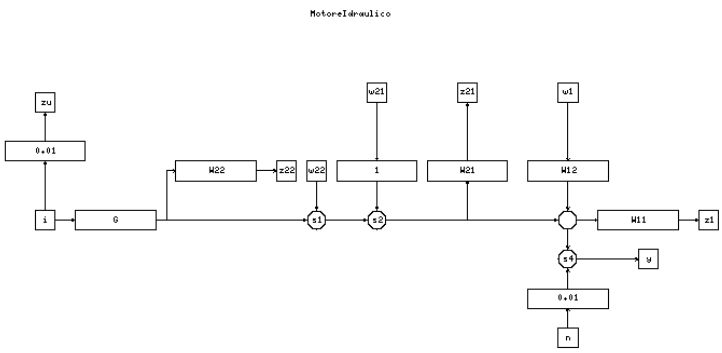

Fig. 9.9 Extended system of the control of an hydraulic motor – case 1

The weighting function (9.10) on the sensitivity has been split in two functions W_{12} e W_{11}. Function W_{11}, in particular, has two poles in zero, for this the extended system is not stabilisable. Situation that requires some caution during the design.
The H_∞ norm of the closed loop operator reach the value of 1.5, technically we are not achieved robust performances. Robust stability and nominal performances, are viceversa, satisfied. To achieve this result constraints on the control activity have been relaxed and as before control activity exits from the requirements.
The next graph shows the achieved results.

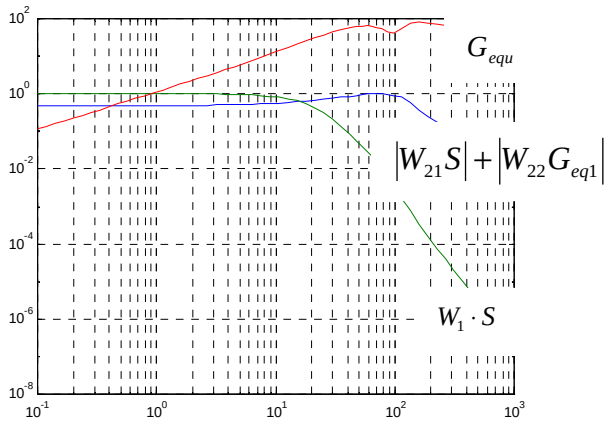

Fig. 9.10 Results of design – case 1

The compensation filter is the following

$$G_c = \frac{22150.1323\,(s+10.35)\,(s^2+27.92s+424.5)\,(s^2+60.85s+9895)\,(s^2+54.1s+1.02e004)}{s\,(s+231.2)\,(s^2+26.71s+379.6)\,(s^2+68.5s+1.011e004)\,(s^2+114.1s+2.009e004)}$$

It is interesting to compare it with the one obtained in the classical design: the pole in zero is present, as well as the lead action. Three pairs of complex conjugate poles have been added: tow pairs can probably be simplified with corresponding zeroes, but the third one has the clear structure of a notch filter to reduce the gain of the loop at the high frequencies.

9.4.2 Model 2

We adopt now the second of the two proposed models, and we add to the uncertainty represented in figure 9.4 requirements on the sensitivity. We found convenient to enforce these requirements introducing an input noise and assigning the weights on the output. The complete representation is given in the next figure

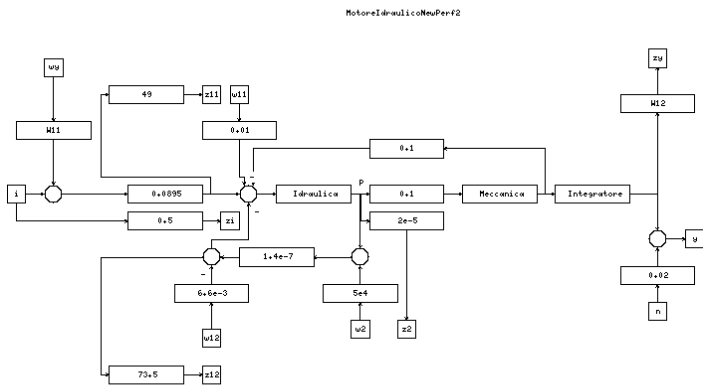

Fig. 9.11 Extended system of the hydraulic motor and its loop requirements

Structured analysis allow to avoid to integrate different requirements in a unique disturbance-objective pair. Viceversa each specification is assigned to one input-output pair. In the function W_{11}, as its objective is contrasted by the input noise, only one pole in zero necessary (the plant has already one pole in zero).
Applying the design process it appear clearly that the uncertainty description is heavier with respect to the previous case. To obtain robust stability we had to relax further control constraints. In this case, also, the best result guarantee H_∞ norm of 1.5. Results are presented in the next figure

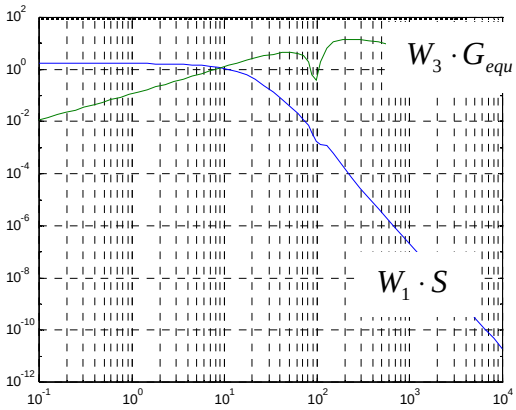

Fig. 9.12 Indices of performance and control activity

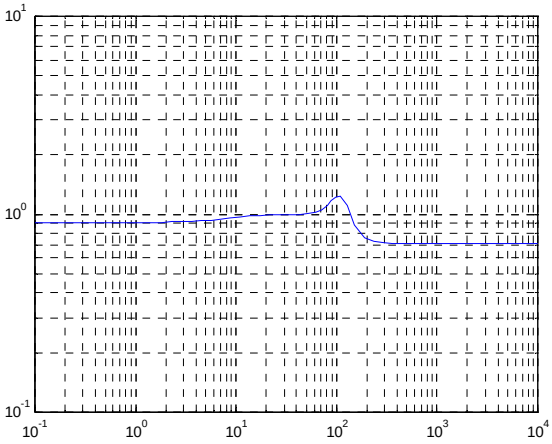

Fig. 9.13 Index of robust stability

The resulting filter is the following

$$G_c = \frac{66795.2251\,(s + 6.675)\,(s^2 + 27.97s + 411.8)\,(s^2 - 0.2509s + 8793)}{s\,(s + 401.7)\,(s^2 + 27.97s + 399.4)\,(s^2 + 185.4s + 3.042e004)} \tag{9.13}$$

In the next figure the filters built with the synthesis are compared in frequency with the classical one.

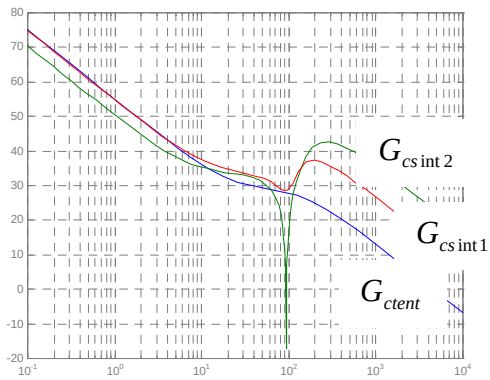

Fig. 9.14 Compensation filters

The comparison evidences some interesting observations. Both results of synthesis have the same structure: pole in zero, lead action and notch filter. Note how the synthesis has

interpreted originally the notch filter, that with poles and zeros slightly separated, one form the other, contributes to increase the phase margin.
We leave as exercise to verify that to completely satisfy requirements, achieving robust performance, is sufficient to relax the condition on the sensitivity reducing the required loop band.

9.4.3 *Tachometric feedback*

In the previous section, measuring the position only, we were close to satisfy all performance and robustness requirements at a cost on the control activity (approximately the norm is 10 times greater than the desired value).
Responsible for this is the lead action need to satisfy the necessary stability margins.
To obviate to this situation it is customary to add to the position the measure of the velocity. Using this signal for control is called *tachometric feedback*. More information on the system state can only be beneficial, especially in reducing the control activity at equal performances, as it will be evidenced in the next example.
The new extended system is

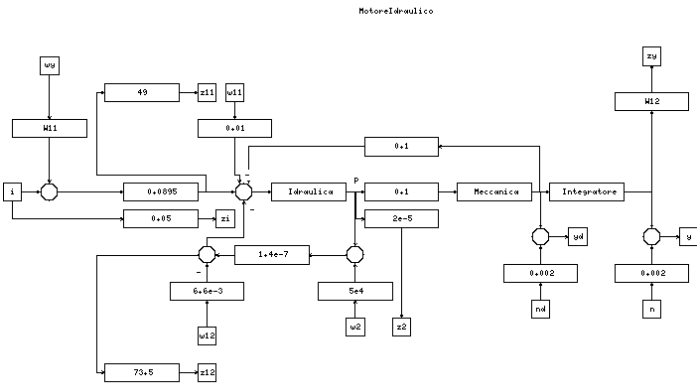

Fig. 9.15 Extended system of the hydraulic motor with tachometric feedback

With this model robust performances are positively achieved. The control filter is the following

$$u = \left[\begin{array}{cccccc|cc} -6.5 & -2.34e8 & 7.54e8 & 2.14e6 & -.35e6 & -2.7e8 & -14e3 & -824e8 \\ 0.004 & -1370 & 4414 & -44 & 93.7 & -1633 & 343.6 & -5408 \\ & & & -15.8 & 31.64 & -14.4 & 161.3 & -113.8 \\ & & & 0.225 & -31.68 & 0.069 & 14.36 & -0.6 \\ & & & 12.83 & -28.22 & -1.16 & 0.41 & 0.698 \\ & & & & & -1 & 148.8 & 148.3 \\ \hline -12.4e-5 & -0.11 & 0.079 & 0.11 & -0.22 & 0.27 & & \end{array}\right] \cdot \begin{bmatrix} y \\ \dot{y} \end{bmatrix} \tag{9.14}$$

It is interesting to see the frequency behavior of the filter if the tachometer feedback is transferred to the output, i.e.:

$$G_c = G_{cy} + G_{c\dot{y}} \cdot s \tag{9.15}$$

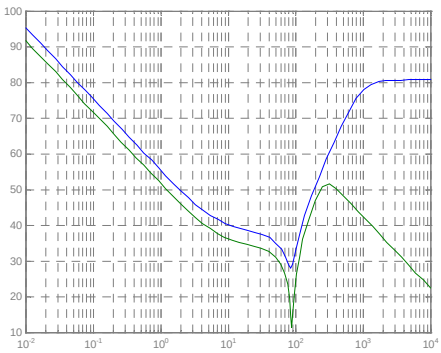

Fig. 9.16 Effect of the tachometer feedback on the compensation filter

Frequency behavior of $\left|W_3 \cdot G_{equ}\right|$ with and without tachometer feedback are compared in the following figure

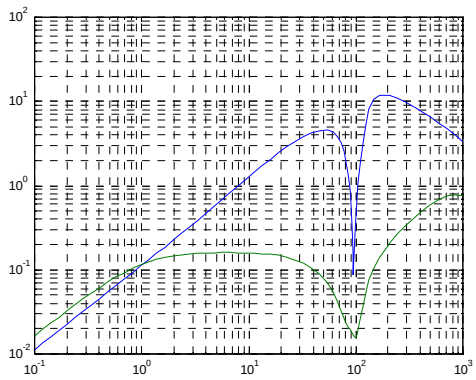

Fig. 9.17 Frequency analysis of $\left|W_3 \cdot G_{equ}\right|$

Indices of nominal performance and robust performance are:

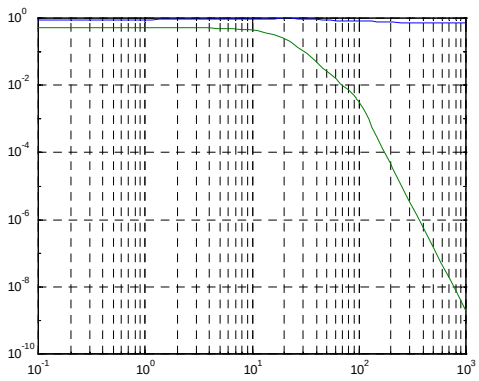

Fig. 9.18 Indices of robust performance $\left|W_1 S\right|, \left\|G_{eq}(1:3,1:3)\right\|$

9.4.4 Two degree of freedom design

We approach the two degree of freedom design, adopting the extended model of figure 9.19.

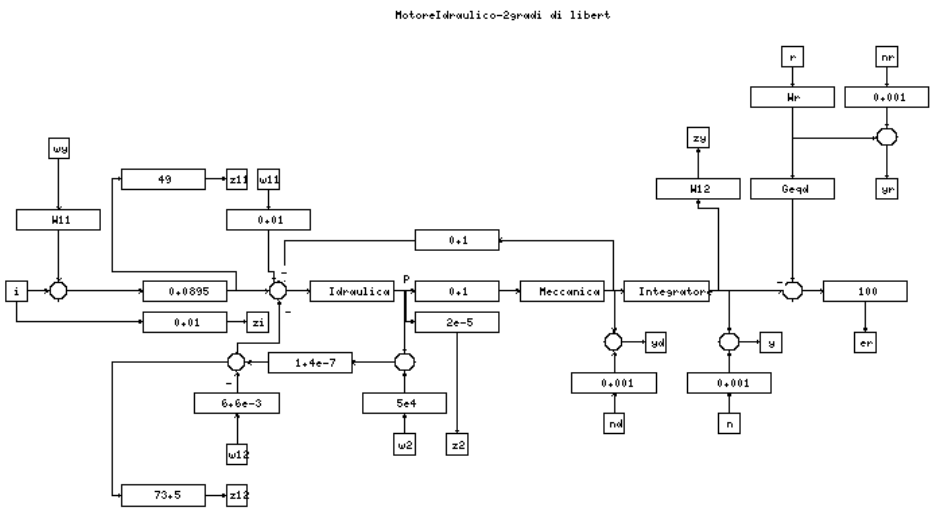

Fig. 9.19 Extended system for two degree of freedom design

To the diagram of figure 9.15 a further objective weighting the error between the output and the reference model G_{eqd} has been added, contrasted by the reference signal. As the reference is know a measurement noise on the reference is present.
To guarantee precise steady state gain the reference has been weighted by an integrator (function W_r). As the objective on tracking influence the sensitivity the gain of W_{12} as been reduced with respect to the previous design to achieve the desired loop band. Also the gains on measurement noise and control have required an adjustment.
The extended system has in the problem two poles in zero uncontrollable.
The desired G_{eqd} is (9.16). Achieved results are detailed in the following

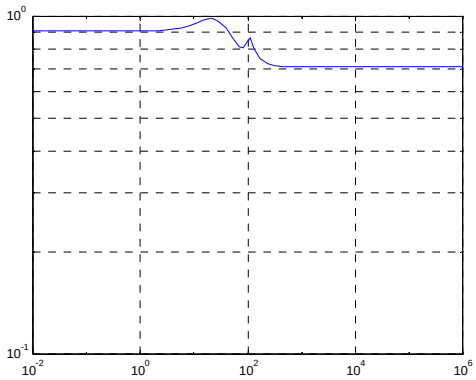

Fig. 9.20 Index of robust stability $\left\| G_{eq}(1:3,1:3) \right\|$

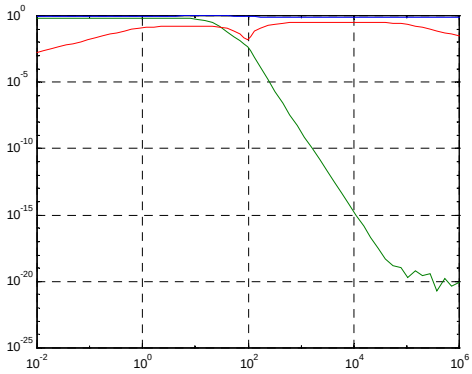

Fig. 9.21 Indices of robust performance, sensitivity and control activity

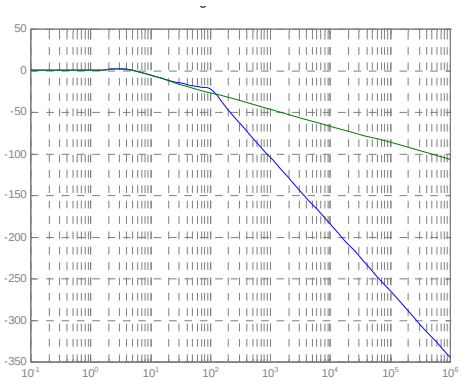

Fig. 9.22 Desired and achieved closed loop transfer functions

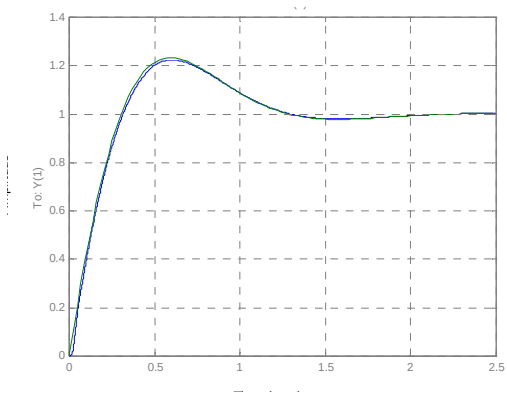

Fig. 9.23 Comparison on a step response

The controller is a dynamical filter of the ninth order with three inputs. Nevertheless, it is interesting translate this filter in the classical structure of a two degree of freedom design with one filter in feedforward and one in cascade. The result is in the following formula

$$G_f = \frac{0.040532\,(s+63.21)\,(s+14.43)\,(s+3.685)\,(s^2+1695s+1.435e006)}{(s+100)\,(s+16.6)\,(s^2+4.8s+16)\,(s^2+30.64s+7359)}, \qquad (9.17)$$

$$G_c = \frac{20066.6\,(s+16.6)\,(s^2+30.64s+7359)}{s\,(s^2+1819s+1.65e006)}$$

9.5 Digital control design

We select the sampling period taking into account that the extended system has an important dynamics at 100 rad/s. We assume for that a sampling period of $0.006s$. Generated the discretized extended system with z.o.h. sampler, we repeat the design procedure without particular differences with respect to the continuous case. The achieved results, also, matches the one we already have.
The closed loop poles in the continuous case were

$$\begin{array}{rr} -847.44926 & 846.6179; \\ -847.44926 & -846.6179; \\ -27.689613 & 98.626545; \\ -27.689613 & -98.626545; \\ -99.999932 & 0; \\ -63.211479 & 0; \\ -23.747982 & 0; \\ -14 & 14.282857; \\ -14 & -14.282857; \\ -13.99999 & 14.282848; \\ -13.99999 & -14.282848; \\ -1.0000001 & 0; \\ -2.4000019 & 3.1999773; \\ -2.4000019 & -3.1999773; \\ -2.4 & 3.2; \\ -2.4 & -3.2; \\ 0 & 0; \\ 0 & 0 \end{array} \tag{9.18}$$

The closed loop poles in the discrete case are

real	imag	mod	sigma	omega
0.916	0.0787	0.919	-14	14.3;
0.916	-0.0787	0.919	-14	-14.3;
-0.856	0	0.856	-25.9	0;
0.0532	0	0.0532	-489	0;
0.703	0.477	0.849	-27.3	99.4;
0.703	-0.477	0.849	-27.3	-99.4;
0.67	0	0.67	-66.6	0;
0.889	0	0.889	-19.6	0;
0.916	0.0787	0.919	-14	14.3;
0.916	-0.0787	0.919	-14	-14.3;
0.554	0	0.554	-98.6	0;
0.994	0	0.994	-0.998	0;
0.986	0.0189	0.986	-2.4	3.2;
0.986	-0.0189	0.986	-2.4	-3.2;
0.986	0.0189	0.986	-2.4	3.2;
0.986	-0.0189	0.986	-2.4	-3.2;
1	0	1	3.13e-009	0;
1	0	1	3.13e-009	0

(9.19)

The comparison shows that with the exclusion of a pair of poles that in the continuous case feedback dragged to a high frequency, the significant poles maintain in the two case corresponding positions.
The poles of the compensation filter in the continuous case are

-909.55345	906.8442;
-909.55345	-906.8442;
-100.00022	0;
2.0300555e-007	0;
-13.99999	14.282847;
-13.99999	-14.282847;
-1.00002	0;
-2.4	3.1999776;
-2.4	-3.1999776

(9.20)

and in the discrete

real	imag	mod	sigma	omega
-0.884731	0	0.884731	-20.411939	0;
0.0212825	0	0.0212825	-641.64502	0;
1	0	1	0	0;
0.9160555	0.078697797	0.91942973	-14.000277	14.283167;
0.9160555	-0.078697797	0.91942973	-14.000277	-14.283167;
0.55358165	0	0.55358165	-98.55767	0;
0.99402935	0	0.99402935	-0.99809104	0;
0.9855215	0.018924324	0.98570318	-2.4000009	3.1999975;
0.9855215	-0.018924324	0.98570318	-2.4000009	-3.1999975

(9.21)

9.6 Conclusions

Classical design has shown that only one lead compensation is not sufficient to completely satisfy specifications in this example of control of an hydraulic motor. The experience conducted with the synthesis confirms that a greater stability margin is needed from the loop. This is achieved with an higher order filter, than a simple derivative compensator.

Starting from a simple mixed sensitivity problem that can be solved with classical loop shaping techniques the examples evolved to a more complex model covering a complete envelop of specifications. This emphasizes the flexibility of the extended system approach to cover a wide range of control requirements.