Chapter 8

DESIGN TECHNIQUES

This chapter discusses the technique needed to construct an extended system and the steps suggested in the design process to reach successfully the achievement of the specifications.
Too often we have seen that robust control is approached as a pure optimization problem without care of realistic specifications or regard to the final results. Our experience has shown that to be successful in synthesizing a good control, robust control theory alone is not sufficient, but a certain number of rules have to be followed in building the extended system, in selecting the weighting functions and in reconciling contradictory requirements.
We collect here our experience and we discuss the achieved results.
The presentation is organized with the following schema

1. Setting the specifications
2. Interaction between observer and feedback
3. Structuring to achieve robust performances
4. Presence of poles in zero in the weights

We maintain an unforgivable classical mentality so we start from the loop design in one degree of freedom then we approach input-output specifications extending the design to two degree of freedom.
We suggest to follow in order the steps outlined in the next sections.

8.1 Setting the specifications

8.1.1 Process disturbances and the loop sensitivity

The starting point of the design is in our opinion the loop characteristics and its insensitivity to disturbances. We suggest to set state disturbance specifications first. If these are not specified it is advisable to add some noise to the control. Ignoring these disturbances may result in a control that doesn't satisfy internal stability conditions.
The objectives on the outputs have the scope to specify insensitivity to disturbances. These requirements affect the function G_{eqd} and indirectly the sensitivity function. So the weights on the objectives must be chosen taking into account the frequency band that realistically we expect to achieve from the loop. If a too large band is assumed on the

objectives it will result in a poor design or not design at all, conflicting this with other specifications.
Sometime, as G_{eqd} hides the direct view of the sensitivity function, and a precise range of the sensitivity is desired, process disturbances directly on the output of the system can be added, as done in all classical control texts. We know that this is unrealistic, nevertheless, (the maximum singular value at each frequency of) the output weighting function, is, in this case, precisely the inverse of (the maximum singular value of) the expected sensitivity. We have found in many cases that a requirement directly on the sensitivity, added to other disturbances rejection specifications, not only doesn't interfere, but also facilitate the achievement of the results.
We are not discussing here the convenience to assign weighting functions to disturbances (modeling disturbance dynamics) or to the objectives (weighting the objectives), or to split weights between them. This problem is identical for any other specification. Let consider that from the point of view of an isolate requirement what matters is the loop that is generated, closing the control, between disturbance and objective, hence the product of the two functions. However, the selection between input and output to assign weights is important for a different reason: the interaction between different specifications, that is strongly influenced by the weighting functions. This topic will be better discussed in the section related to structuring.

8.1.2 Coping with uncertainty

Modeling uncertainty, as before, influences the loop characteristics, however, at difference to disturbance sensitivity specifications, uncertainty may involve any one of the four closed loop characteristic transfer functions, and not just G_{eqd} or S (see section 5.1.3 of chapter 5).
A few rules have proven to be effective in the design.
Don't forget, among others, a steady state loop gain uncertainty. The loop gain is anyway expected somehow to change and it is easy to understand, as it imposes classical loop gain stability margins.
We all know that the loop performance cannot extend beyond a certain frequency. Growing uncertainty on the system dynamics from a frequency on is useful to constraint the realistic achievable loop band.
Both aspects involve a multiplicative form of description of the uncertainty and affect the complementary sensitivity function G_{eq1}. They ca be easily merged in a unique function as shown in the next figure.

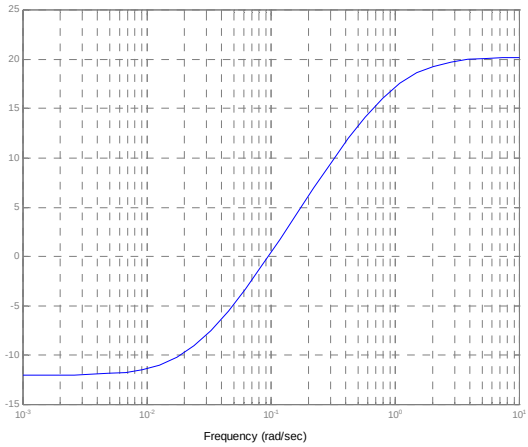

Fig. 8.1 Multiplicative uncertainty covering steady state gain and high frequency

Let consider that uncertainty, as in figure 8.1, joint to a performance on sensitivity as described in the previous section constitutes an essential and yet complete specification for a well posed elemental closed loop design. The resulting problem is called in the literature mixed complementarily control problem.

8.1.3 Measurement noise and control activity

There are two main reasons to fear control activity: fast reference changes in tracking and measurement noises, and invariably their effects involve the high frequency spectrum.

For this reason a penalty on the control activity in term of pure gain on the objective and noise on measures and reference spanning throughout the whole frequency range are required. It is immaterial that these requirements answer, also, to technical reasons related to the gramian minimization[19]. We found control weights and noise gains a simple and invaluable tool to control how tight (gain and frequency band) we want to close the loop.

We suggest to start the design with relatively small gains[20]. In these conditions we leave to the control the complete freedom to satisfy all other specifications. Ignore the control activity, at first, and look at the results for all other requirements. If they are not fully satisfied evidently they interfere each other (e.g. the band required from the sensitivity interferes with the presence of uncertainty at high frequency) and it is necessary to relax some of them.

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[19] Without these two element the quadratic optimization problem become singular

[20] Too small gains, at the limit of the singularity of the optimization problem can be easily recognized by some of the closed loop poles moving to high frequency and magnitude of feedback gains growing irrealistically.

When all other specifications are satisfied we start looking at the control activity. Then the design is repeated growing the gains (control weight and noise level) until also the control performances enter into the specifications, or again some of the requirements have to be relaxed.

Multiple measures

Potentially, more measures are available on the system better are the performances that can be achieved from the control. The clear evidence of this classical statement can be seen if the same specifications are achieved with different controllers using one or more output measures (position, velocity, acceleration,..). You will experience with a proper exploitation of each measure, and similar achieved performances, a reduction of the control activity at the growth of their number.
However, this is not always the case if measures are not exploited correctly. The reason is that the same or similar closed loop characteristics in the multivariable case can be achieved with different gains of each measure channel. A global reduction of the control activity is found by balancing the contributions that all channels give to the feedback. This can be obtained recursively playing on the relative measure noise gains channel per channel.

8.1.4 Reference-output and two degree of freedom

We suggest to review figure 6.38 of chapter 6 before approaching this section. One justification for the two degree of freedom design is to offer insensitivity performances to the closed loop reference-output operator.
This observation will drive the design technique we propose here. Let start with one degree of freedom design satisfying all loop specifications, hence also sensitivity.

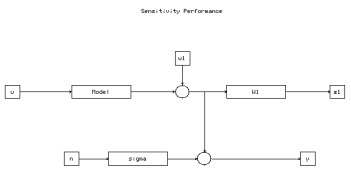

Fig. 8.2 One degree of freedom

Then add to the extended system the elements needed to approach a two degree of freedom design. Leave the output weighting function unchanged (actually the function that helped to shape the sensitivity in the previous design step) but simply shift the objective from the output to the output tracking error.

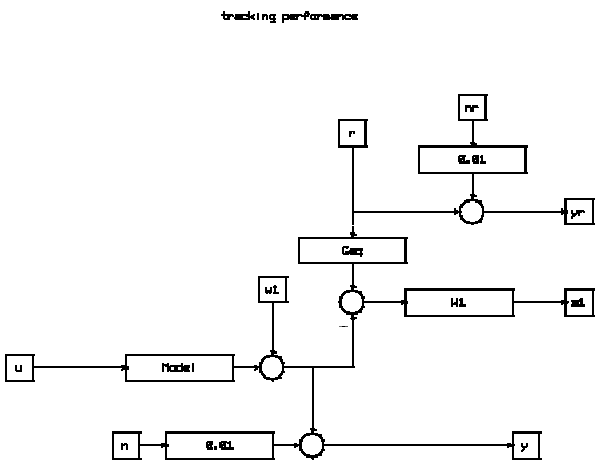

Fig. 8.3 Two degree of freedom

Repeating the design, the loop characteristics will be maintained, contemporaneously the desired closed loop transfer function will be matched with an accuracy in frequency directly related to the sensitivity function. In fact, the attempt to achieve a better accuracy over a larger frequency band would be completely useless given the uncertainty.
More precisely, with this specification, we will guarantee the norm of the absolute difference between reference and achieved Geq [21]

$$\left\| \Delta_{Geq} W_1 \right\|_\infty \leq 1 \tag{8.1}$$

for any admissible uncertainty.

8.2 Interaction between observer and feedback

We approach control design in two steps: to facilitate understanding observer design in the first step and then state feedback. This sequence allow us to introduce our idea of the black box model. However, by duality, the sequence of the two steps can be interchanged. The controls resulting in the two cases, even if not identical, are equivalent from the point of view of the performances so the next considerations are applicable to both orders of sequence.

[21] Note that in classical design the sensitivity function guarantees the relative variations , not the absolute one.

Start each step by minimizing the gramian, i.e. the H_2 norm of the closed loop error operator. Evaluating in this condition the achieved H_∞ norm will give you immediate filling of how far you are from satisfying your requirements. Assume this value as the bound for a new design obtained, this time, by guaranteeing the upper bound of the H_∞ norm. In turn, assume the achieved norm value, usually smaller than the bound, as a new bound to repeat the previous step recursively.
The process will eventually bring you toward the minimum achievable H_∞ norm. However, in general, but especially if this is the first step you don't need to go too far in the process, choosing a too low upper bound value. In fact, this bound has to be satisfied also by the state feedback of the black box model, performed in the second step, to guarantee final output feedback performances.
If in the state feedback design you are not able to reach the bound satisfied by the observer error you have to design a new observer with an higher upper bound.
If you analyze the interaction between observer and control you will generally experience the following fact: "*lower is the bound selected for the observer, higher is the minimum norm that you can reach with the control*". With this in mind you will be able to renegotiate the observer bound until this bound is satisfied also by the control. This value is proven by H_∞ control theory to be the guaranteed bound of your final performance.
However, the norm that actually is achieved by the closed loop system may be far better than the bound jointly satisfied by observer error and state feedback control.
From this point of view we found that the robust control design is a robust approach in the sense that we illustrate next.
We know that the control structure achieved by guaranteeing the H_∞ norm doesn't satisfy the separation theorem. This means, as example, that the final closed loop poles differ from the poles found separately during the observer error, and the state feedback steps. This is a disadvantage with respect to the separation case as the shift of the closed loop poles is unpredictable and it will be discover only when the final control is applied to the model.
A little though indicates that the gain coupling observer and feedback dynamics and causing this closed loop pole shifting is a function of the value of the guaranteed upper bound: lower is the bound selected, stronger is the coupling and viceversa higher is the bound and closer you are to the separation results.
So, assume to choose a relative high value of the upper bound for guaranteeing observer error, eventually much higher than the norms that you can achieve with observer error and state feedback operators. Evidence shows that very often the final performances disregard the guaranteed upper bound, and instead is close to the worst between observer error and state feedback norms values achieved in the two design steps. This is confirmed by theoretical results discussed in the appendix A1.
Previous observations suggest the following design rules: try to reach your specifications fixing a relative high upper bound during the design, observer and (in particular) feedback partial results will be realistic indicators of the final achieved performances. Your control structure will be close to separation still satisfying the H_∞ norm value you have

chosen for your requirements. Resort to the classical robust control design by reducing, eventually to the minimum, the guaranteed upper bound only in the cases that cannot be solved otherwise.

8.3 Structuring for robust performances

Trying to reach the desired H_∞ norm bound less or equal one for the closed loop extended system, as it is after setting all specifications, rarely will result in a good design. The control, in fact, is too conservative, as different specifications and uncertainties interact each other. This is clarified by figures 5.16 and 5.17 and by the related discussion in chapter 5.

We have seen that structuring the closed loop operator is necessary for obtaining meaningful results. This means designing a control based on the maximum structured singular value and guaranteeing robust performances for each objective. If you think to the properties of structuring this means isolating groups of pairs disturbances-objectives from each other.

We suggest to adopt structuring not only to isolate independent sources of uncertainty, and uncertainty from performances, but also performances from themselves.

We know that minimizing the maximum singular value, structuring the extended system, reaches the maximum structured singular value and offers a necessary, as well as sufficient condition for robust performances, only if the number of structure blocks is less than or equal to three. In all other cases we obtain only an upper bound and the resulting condition is only sufficient.

Nevertheless we suggest to adopt structuring techniques integrally, separating each requirement and each source of uncertainty. Even if it is not strictly needed, in order to facilitate the implementation of structuring we always impose an equal number of disturbances and objectives, adding or replicating an objective or a disturbance any time is needed. At the contrary to the appearance, doing so doesn't cause over-weighting of the objectives, as structuring will take care automatically to maintain the correct interpretation of each requirement.

In practical problems the number of blocks may in the order of tents. In spite of that, we always found remarkable results, and we never had to experience a poor design for too conservative specifications, as when no structure is adopted.

The next figure, to compare with 5.16, explains previous idea.

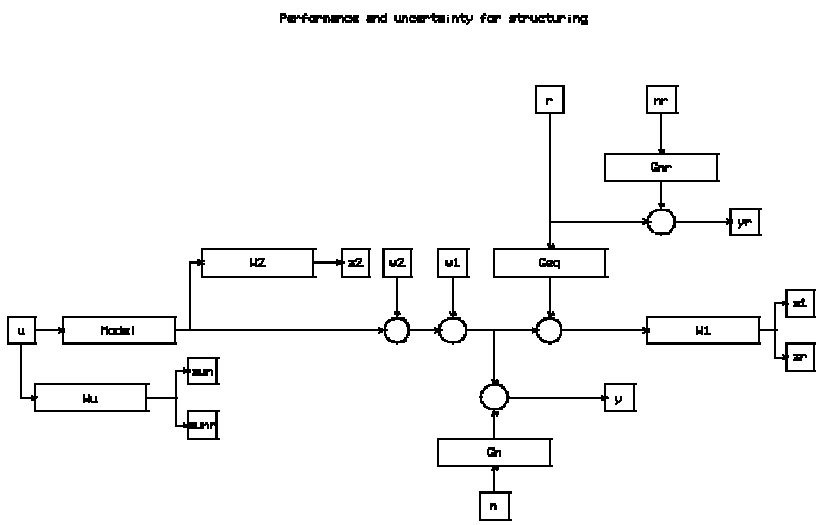

Fig. 8.4 Two degree of freedom prepared for structuring

In conclusion, to verify if the requirements are reachable the control has to be computed in the presence of the best structuring. In general performing the design only once will not be sufficient, instead, the control will be obtained at the end of a process obtained by repeating several design sessions recursively, adopting at each new session the best structuring matrix of the previous one.

8.3.1 Computing the structuring D matrix

Structuring the extended system should start from the decision on the best splitting of the weights on the input or output of the system. However we haven't found so far a conclusive answer to this problem. In theory the structuring matrix *D* can be any dynamical operator and the process of structuring should accomplish the task of distributing the weights automatically, irrespective of the initial choice of the designer. In practice, however, implementing this approach goes too far in complexity.
Structured analysis means computing the best values for the matrix *D* as function of the frequency, and then adopting for *D* a dynamical operator that interpolates these values.
For parsimony, we have decided to adopt for *D* a constant matrix, without performing frequency analysis and where each element of *D* is computed with the procedure (5.43) described at the end of chapter 5, where each norm present is the H_∞ norm of the corresponding operator. We refer that this simplification has never reduced the effectiveness of the approach.

8.4 Presence of poles in zero in the specifications

Until recently we were unable to cope with unstable poles in the weights adopting a robust control approach, as the basic requirements of H_∞ theory is a stabilisable and detectable extended system.
A recent result by Mita, Xin and Anderson [1] has open a completely new perspective.
The idea is shown in the following figure

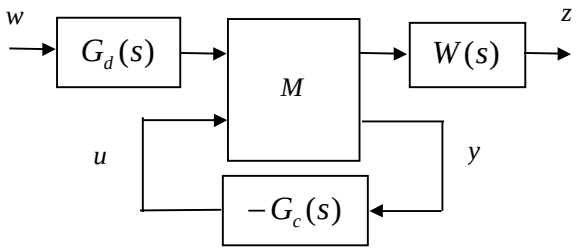

Fig. 8.5 Extended system with unstable weights

Assume to have unstable poles in G_d and/or W and to design a control that guarantees internal stability of the closed loop model inside the extended system and cancel with zeroes the unstable poles present in the weights, that differently could not be influenced by the feedback.
You may wonder that we lack of internal stability in the whole extended system, but this may not be a problem as the weights will not be present in the real plant. The important fact is that the extended system in closed loop satisfies the specifications in frequency, and this is ensured by the cancellations.
The situation where internal stability of the closed loop system, except for the weights, is verified has been called essential stability. The situation where, in adjoint to the essential stability, a cancellation between unstable zeroes and poles is obtained has been called comprehensive stability in [2].
It has been demonstrated that essential stability can be obtained by adopting a particular state space transformation that isolates detectable and stabilisable components of the extended system and by performing the design on these components, only.
However, in order to guarantee also comprehensive stability (i.e. the cancellation between unstable zeroes and poles) geometric conditions imposing, among others, orthogonality between certain vectorial subspaces, have to be satisfied by the model.
These conditions depend on the control problem (model, control requirements, choice of the weights), cannot be designed, but simply verified. Moreover, perfect cancellation, as well as these geometric conditions, as they involve orthogonality, can never be exactly satisfied in practice because of numerical approximations. Nevertheless, we have noted that closer the conditions are to be satisfied, according to a certain measure, closer we

are to a cancellation, and also, that this result is strongly influenced by the location of weights and the structuring we apply to the extended system.
In practical design problems we never encountered the need to introduce purely unstable poles in the weights, poles in zero, viceversa, appear quite often.
This is a special situation for two reasons:

1. We have to detect the poles in zero, e.g. discriminating them from some very low frequency poles sometime present in the specifications;
2. The gain in frequency of the extended system goes to infinite at steady state.

The first of the two previous aspects requires to adopt a criteria or a tolerance for detection, considering that the result of this detection will affect the state space decomposition and hence the design.
The second aspect, linked more generally to the stability of the extended system in closed loop, involves the H_∞ norm computation. Strictly speaking doesn't exist a norm value, as the system is formally unstable. However, we have to consider the cancellation or we have to force it in order to correctly interpret the results. In turn, the norm computation is critically needed to perform structuring (see (5.43) of chapter 5).
These topics are detailed in the next two sections.

8.4.1 Detecting unstabilizable/undetectable poles in zero

To detect unstabilizable/undetectable dynamics, as well as to detect the presence of poles in zero, we have to resort to tolerance thresholds. Our experience suggests to maintain two distinct values, one for each detection.
These thresholds will be played by the designer to force the achievements of the expected results, exploiting also the knowledge he has of the problem.
Some advises may be useful.
With an approach to design performed in two steps, the first step, or both steps when we adopt the separation between observer and control, presents usually little detection problems. In the second step, viceversa, when guaranteed H_∞ norm bound is adopted, because of numerical problems related to the manipulation of the dynamics in building the black box model ((7.36) of chapter 7), detection is more difficult, and raising the threshold value may be necessary. Again, the knowledge that the designer has of the structure of his problem will be important to achieve the correct results.
At the end of each detection an evaluation of the criteria that guarantees cancellation will be important for monitoring the design process.
At the final step of the design, when the controller is synthesized, the same threshold used to detect poles in zero in the weights will play a second important role. Often, the cancellation in the closed loop extended system asks for one or more poles in zero in the controller. Because of numerical problems they may not be immediately evident from the results. It will be, again, responsibility of the designer to force those candidate poles of the controller to the origin. This will be achieved using the same technique of detection we have illustrated before.

8.4.2 Norm computation in the presence of unstable poles cancellation

Once a design has been completed, the H_∞ norms of several restrictions of the extended system in closed loop have to be computed. These are needed to structure the closed loop operator for robust performance analysis. Obviously, algebraic techniques for norm computation fail as we said before, we have to resort to evaluate norms numerically by searching for the maximum in frequency of the maximum singular value of the matrix.
Invariably, with poles in zeros and not perfect cancellation, the maximum of some of the restrictions will be located at the lowest extreme of the frequency range, that must be set greater than zero if you don't want that the norm goes to infinite.
Structuring, in turn, minimizes the operator norm as a function of the diagonal structuring matrix *D*.
If this process is performed recursively, as suggested in the previous section, adopting at each iteration the best structuring matrix of the previous iteration for performing a next design session you will experience that the test for the conditions of cancellation will improve, and the cancellation will get closer from one session to the next.
In conclusion a recursive design process with structuring will actually find the best structuring needed for improving satisfaction of the comprehensive stability condition.

8.5 Bibliography

[1] Mita, T., Xin X., B.D.O. Anderson 2000, *Extended H_∞ control -- H_∞ control with unstable weights.* Automatica n. 36 pp 735-741 (2000).

[2] Liu, K. Z., Zahng, H., Mita, T, *H_2 optimal control under comprehensive stability*. Proceedings of the 34th CDC pp 4008-4113 (1995).